Exercises

1. Which of these sentences are propositions? What are the truth values of those that are propositions?
   a) Boston is the capital of Massachusetts.
   b) Miami is the capital of Florida.
   c) $2+3=5$.
   d) $5+7=10$.
   e) $x+2=11$.
   f) Answer this question.

2. Which of these are propositions? What are the truth values of those that are propositions?
   a) Do not pass go.
   b) What time is it?
   c) There are no black flies in Maine.
   d) $4+x=5$.
   e) The moon is made of green cheese.
   f) $2^n \geq 100$.

3. What is the negation of each of these propositions?
   a) Linda is younger than Sanjay.
   b) Mei makes more money than Isabella.
   c) Moshe is taller than Monica.
   d) Abby is richer than Ricardo.

38. Construct a truth table for each of these compound propositions.

a) $(p \vee q) \vee r$

b) $(p \vee q) \wedge r$

c) $(p \wedge q) \vee r$

d) $(p \wedge q) \wedge r$

e) $(p \vee q) \wedge \neg r$

f) $(p \wedge q) \vee \neg r$

39. Construct a truth table for each of these compound propositions.

a) $p \rightarrow (\neg q \vee r)$

b) $\neg p \rightarrow (q \rightarrow r)$

c) $(p \rightarrow q) \vee (\neg p \rightarrow r)$

d) $(p \rightarrow q) \wedge (\neg p \rightarrow r)$

e) $(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$

f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

11. Let $P(x)$ be the statement "$x = x^2$." If the domain consists of the integers, what are these truth values?

a) $P(0)$ **b)** $P(1)$ **c)** $P(2)$

d) $P(-1)$ **e)** $\exists x P(x)$ **f)** $\forall x P(x)$

12. Let $Q(x)$ be the statement "$x + 1 > 2x$." If the domain consists of all integers, what are these truth values?

a) $Q(0)$ **b)** $Q(-1)$ **c)** $Q(1)$

d) $\exists x Q(x)$ **e)** $\forall x Q(x)$ **f)** $\exists x \neg Q(x)$

g) $\forall x \neg Q(x)$

is in lowest terms. Obtain an equation involving integers by multiplying by b^3. Then look at whether a and b are each odd or even.]

28. Prove that if n is a positive integer, then n is even if and only if $7n + 4$ is even.

29. Prove that if n is a positive integer, then n is odd if and only if $5n + 6$ is odd.

30. Prove that $m^2 = n^2$ if and only if $m = n$ or $m = -n$.

31. Prove or disprove that if m and n are integers such that $mn = 1$, then either $m = 1$ and $n = 1$, or else $m = -1$ and $n = -1$.

32. Show that these three statements are equivalent, where a and b are real numbers: (*i*) a is less than b, (*ii*) the average of a and b is greater than a, and (*iii*) the average of a and b is less than b.

14. Prove that if x is rational and $x \neq 0$, then $1/x$ is rational.

15. Prove that if x is an irrational number and $x > 0$, then $\sqrt{x}$ is also irrational.

16. Prove that if x, y, and z are integers and $x + y + z$ is odd, then at least one of x, y, and z is odd.

17. Use a proof by contraposition to show that if $x + y \geq 2$, where x and y are real numbers, then $x \geq 1$ or $y \geq 1$.

18. Prove that if m and n are integers and mn is even, then m is even or n is even.

19. Show that if n is an integer and $n^3 + 5$ is odd, then n is even using
    a) a proof by contraposition.
    b) a proof by contradiction.

20. Prove that if n is an integer and $3n + 2$ is even, then n is even using
    a) a proof by contraposition.
    b) a proof by contradiction.